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Load Carrying Capacity of Concrete Deck Slabs

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The results of the test show that conventionally reinforced deck slabs have very high factors of safety against failure by punching and are wastefully over-reinforced.

Key Words: punching failure, isotrophic reinforcement, orthotropic reinforcement, reinforced deck slabs, composite I-beam bridges

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A theory, based on a mechanical model of punching failure of simply supported slabs, is developed which permits the calculation of the punching strength of restrained slabs. The theory suggests that a deck slab can be expected to have a high inherent strength due to boundary restraints ensured by the presence of shear connectors, beams, diaphragms and the neighbouring slab areas.

One eighth scale direct models of 80 ft. (24.4 m) span prototype bridges were tested in laboratory studies of both orthotropically and isotropically reinforced slabs. Shear connector behaviour and dead load stresses appropriate to unshored construction were simulated.

The results of the tests show that conventionally reinforced deck slabs have very high factors of safety against failure by punching and are wastefully over-reinforced.

From considerations of ultimate strength as well as shrinkage and temperature reinforcement requirements, 0.2% isotropic reinforcement is recommended as being adequate for bridge slabs of the type studied. Although this amounts to approximately 30% of the current reinforcement requirements for such slabs, a high factor of safety can still be expected.

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1/Notation

A_s	area of reinforcement per unit width of slab
B	diameter of loaded area
C	diameter or equivalent diameter of slab
C_b	maximum compressive force in concrete per unit length of boundary
d	effective depth of slab
F_b	boundary force per unit length of boundary
$F_{b\ (max)}$	maximum boundary force
F_r	restraint factor
f_{max}	maximum stress in concrete
f_{sy}	yield stress of reinforcement
f'_c	compressive strength of concrete
k	ratio of average stress to maximum stress
M_b	boundary moment per unit length of boundary
$M_{b\ (max)}$	maximum boundary moment
P	theoretical punching load without correction for dowel and tensile membrane effects
p	reinforcement ratio (A_s/d)
q	reinforcement index (pf_{sy}/f'_c)
R_1, R_2, R_3	in-plane resultant forces at punching failure
T	oblique compression force in conical shell
T_b	maximum force in tensile reinforcement per unit length of boundary
t	thickness of slab
V	punching load
Δ	deflection at punching failure
β	centre angle of sector element
γ	flexural factor = $V_{flex}/bd\ f'_c$ where b = perimeter of loaded area and V_{flex} = ultimate flexural capacity

2/Introduction

The research described in this report was undertaken to investigate the ultimate strength of the concrete deck slabs of composite steel/concrete bridges under concentrated loads. It was thought that the inherent restraint of slabs of structures of this form could result in enhancement of load-carrying capacity and if this were considered in design, the reinforcement of the slab could be reduced. This report is a summary of an extensive investigation carried out at Queen's University (Hewitt, 1972).

It is current practice (AASHTO, 1973; CSA, 1974) to design the deck slab of a composite bridge by assuming a wheel load to be distributed over transverse slab strips of unit width that are perpendicular to the direction of the traffic. After approximating the maximum moments to be resisted, the slab strips are designed using the traditional elastic methods, as if they were concrete beams. It is understood that slabs designed in this manner can be regarded as being safe against shear type failures.

The investigation sponsored by the Ontario Ministry of Transportation and Communications under Project Q-50 has confirmed that the conventional design of deck slabs of composite bridges is very conservative, and that even with considerable reduction of the slab reinforcement, satisfactory factors of safety against failure by punching can be anticipated. Similar high strengths for restrained slabs have been noted by several including Taylor and Hayes (1965) and Batchelor and Tong (1970).

This report shows that by considering the mechanism of failure, an adequate and more economical slab design can be achieved.

3/Outline of Theory

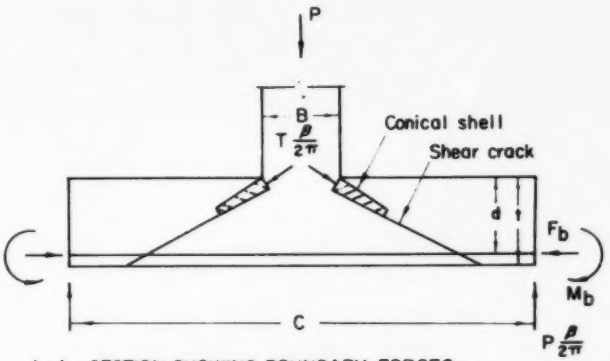
The idealized model of failure proposed by Kinnunen and Nylander (1960) has been proven to permit a good estimate of the punching strength of simply supported slabs. The punching shear strength of restrained slabs has been investigated (Hewitt 1972) by modifying Kinnunen and Nylander's theory to incorporate a boundary restraining force F_b and a boundary restraining moment M_b. Both F_b and M_b are per unit length of slab and act at the level of the tensile reinforcement at the boundary. The idealized mechanical model adopted for a restrained slab at punching failure is shown in Figure 1.

The outer portion of the slab, which is bounded by the shear crack and by radial cracks, is considered to be loaded through a compressed conical shell that develops from the perimeter of the loaded area to the root of the shear crack. The conical shell is assumed to have the shape shown in Figure 1a, and its thickness is assumed to vary in such a manner that the compressive stresses at the intersection with the column and at the root of the shear crack are approximately equal.

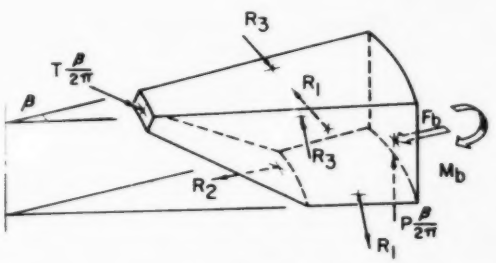
The sector element shown in Figure 1b is acted upon by the external load $P\beta/2\pi$ and by the following forces which are caused by rotation:

1/ The oblique compression force $T\beta/2\pi$ in the compressed conical shell.
2/ Horizontal forces in the reinforcement at right angles to the radial cracks, with resultants R_1.
3/ Horizontal forces in the reinforcement crossing the shear crack, with resultants R_2.
4/ Horizontal tangential compressive forces in the concrete, with resultants R_3.
5/ The boundary restraints F_b and M_b acting at the level of the tensile reinforcement at the boundary.

The criterion of failure is that punching occurs when the tangential strain at the top surface of the slab in the vicinity of the root of the shear crack reaches a critical value. By considering the equilibrium of the sector element and adopting the empirical criterion of failure used by Kinnunen and Nylander, the theoretical punching load P can be determined in an iterative process using a computer program developed for this purpose. The criterion of failure assumes that punching occurs when the tangential strain at the top surface of the slab in the vicinity of the root of the shear crack reaches a critical value. The theoretical punching load can then be corrected for the dowel effect to give V, the ultimate punching load of the restrained slab.



(a) SECTION SHOWING BOUNDARY FORCES



(b) SECTOR ELEMENT SHOWING SLAB FORCES

Figure 1/ Mechanical Model of a Slab with Boundary Restraints at Punching Failure

During the analysis of reported shear tests, it became apparent that the theory by Kinnunen and Nylander was unreliable for estimating the punching shear strengths of slabs of extreme dimensions and material properties. It was found that, for slabs with any of the parameter functions outside the following limits, the theory generally gave inaccurate estimates of the punching load or because of limited test results, the theory could not be verified. These limits are:

Parameter function	Limits
C/d	4 – 17
q	0.05 – 0.45
E_{steel}	$27 \times 10^6 - 33 \times 10^6$ psi
	$(186 \times 10^6 - 227 \times 10^6$ kPa)
γ	4 – 20

The equations used in computing the theoretical punching load are derived and presented in Appendix A.

3.1/ Influence of Boundary Restraints

A hypothetical slab was analysed in order to demonstrate the influence of variations of the boundary restraints on the punching load of a restrained slab. The assumed dimensions and properties of the slab were as follows:

Slab thickness (t)	7 in. (178 mm)
Slab effective depth (d)	5.5 in. (140 mm)
Diameter of the loaded area (B)	12 in. (305 mm)
Slab diameter (C)	72 in. (1.83 m)
Reinforcement (p)	1.0%
Yield stress of reinforcement (f_{sy})	45 ksi (310 MPa)
Compressive strength of concrete (f'_c)	5.0 ksi (34.5 MPa)

The boundary force was varied from 0 to 8,000 lb/in. (0 to 1400 kN/m) and the boundary moment from 0 to 12,000 in. lb./in. (0 to 54 kN·m/m). The theoretical punching load is plotted against the boundary force for various boundary moments in Figure 2. The theory evidently suggests that considerable increase of the punching shear strength of a slab can result from boundary restraining forces.

Few punching tests of restrained slabs have been reported in which the magnitudes of the restraining forces were known or could be inferred. However, tests of prestressed concrete slabs by Scordelis, Lin and May (1958) have permitted a limited investigation of the accuracy of the punching load calculated using the theory proposed for restrained slabs. Seven prestressed concrete slabs were analysed assuming the prestressing cable forces to be boundary restraining forces acting in the plane of the slab. Estimates of the dowel and tensile membrane effects were made for each slab and the influence of the unbonded prestressing cables was considered. The ratio of the test load to the calculated load was found to have a mean value of 1.01 and a standard deviation of 0.044. Considering the assumptions made in estimating the dowel and tensile membrane effects and in assessing the influence of the unbonded prestressing cables, the calculated punching loads were remarkably consistent and accurate. Using an empirical approach, Scordelis, Lin and May (1958) obtained ratios with a mean value of 1.02 and a standard deviation of 0.075. A more in-depth comparison is given by Hewitt and Batchelor (1975). Additional comparisons of tests by others are provided by Hewitt (1972).

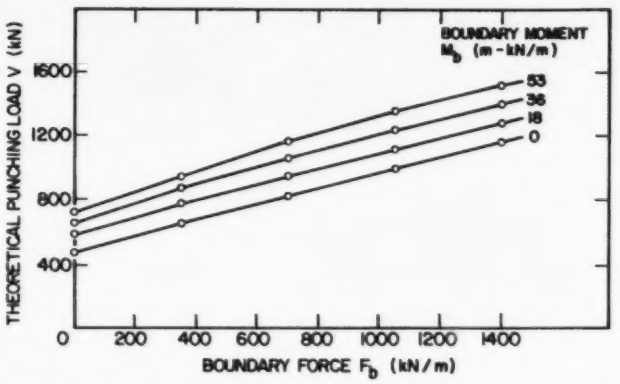


Figure 2/ Variation of Theoretical Punching Load with Edge Restraints

3.2/ Restraint Factor

In practical situations, the boundary restraining forces on a slab loaded to punching failure are usually not known and would be difficult to calculate or to measure accurately. In a slab such as the deck slab of a composite bridge, the difficulty is further increased by the likelihood of varying support and boundary restraint conditions at adjacent boundaries. For these reasons, a single factor F_r, to be termed the 'restraint factor', is proposed which expresses the slab strength enhancement due to practical boundary conditions. This factor can be used for the calculation of the punching load of restrained slabs by means of the largely rational approach already described, though it must be noted that the factor is itself empirical.

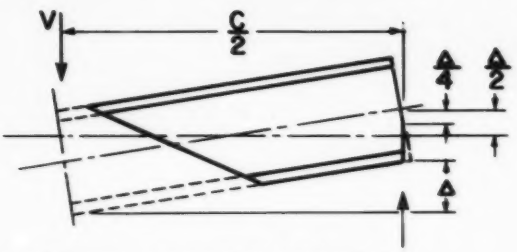
The idealized geometry of displacement of a slab at failure, as used by Brotchie and Holley (1971), and the resultant maximum boundary stresses and forces are given in Figure 3. The maximum force T_b per unit length of the boundary in the tensile reinforcement at the boundary is given by:

$$T_b = d\,p\,f_{sy} \qquad (1)$$

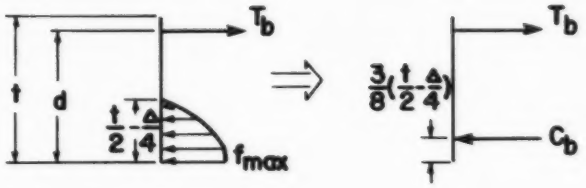
T_b is zero if the reinforcement is not restrained at the boundary or if there is no tensile reinforcement at the boundary. The maximum compressive force C_b per unit length of the boundary in the concrete is given by:

$$D_b = k\,f_{max}\left(\frac{t}{2} - \frac{\Delta}{4}\right) \qquad (2)$$

in which k is the ratio of the average stress to the maximum stress and depends on the stress distribution. If a parabolic distribution of stress is assumed, then k



(a) GEOMETRY OF DISPLACEMENT



$T_b = d.p.f_{sy}$ kN/m

$C_b = k.f_{max} x(\frac{t}{2} - \frac{\Delta}{4})$ kN/m

(b) ASSUMED MAXIMUM BOUNDARY STRESSES AND FORCES

Figure 3/ Idealized Displacement and Maximum Boundary Forces for a Restrained Slab

is 2/3 and f_{max} is assumed to be 0.85 f'_c. The idealized maximum boundary restraints are then given by:

$$M_{b\ (max)} = T_b\ (2d - t) - C_b\ (d - \frac{13}{16}t - \frac{3}{32}\Delta) \quad (3)$$

and

$$F_{b\ (max)} = C_b - T_b \quad (4)$$

Realizing that the maximum boundary restraints would rarely be attained at failure, a value of $F_r \leq 1.0$ was introduced. The actual boundary restraints at failure by punching are then given by:

$$M_b = F_r \cdot M_{b\ (max)} \quad (5)$$

and

$$F_b = F_f \cdot F_{b\ (max)} \quad (6)$$

It is not implied here that the distribution of stress at the boundaries or the actual boundary restraints at the instant of punching are known. It is a fact, however, that F_r varies from zero for the simply supported slabs usually tested in investigations of punching, to unity for slabs with a theoretical, absolute restraint. For this reason, F_r must lie between 0 and 1 for all practical cases of restrained slabs. The restraint factor will depend on the properties of the slab as well as on the confining structure and can be determined empirically for a particular slab system.

After calculating $M_{b\ (max)}$ and $F_{b\ (max)}$, F_r can be determined for any type of restrained slab for which the punching load is known. The lower limit of this factor can then be used to determine punching loads for design purposes. The values of $M_{b\ (max)}$ and $F_{b\ (max)}$ are dependent upon the deflection Δ, and consequently must be calculated iteratively. A computer program has been developed to calculate the punching load of restraint slabs for values of F_r varying from zero to unity in steps of 0.25 (Appendix B).

3.3/ Influence of Restraint Factor

The typical slab previously described was analysed to demonstrate the influence of the restraint factor on the punching load of a slab. The calculated punching load is plotted against the restraint factor in Figure 4. The boundary moment and force for each restraint factor are also given. By varying the restraint factor from 0 to 1, the calculated punching load increases through the full range of feasible punching loads even though actual values of the boundary moment and force may not be used.

If the punching load of a restrained slab is known, the restraint factor can be determined by interpolation after calculating the punching load for given values of the restraint factor. Slabs tested by Taylor and Hayes (1965), Aoki and Seki (1971) and others have been investigated similarly. It was found that the restraint factor of a slab with no tension reinforcement at the boundary generally decreases as the reinforcement index q increases. As a rule, with all variables constant except the reinforcing ratio, a slab with low reinforcement ratio will have deflected more at failure than one having a high reinforcement ratio. Consequently, the boundary restraining forces, which are dependent upon the slab deflection, are likely to be nearer their maximum values in the restrained slab with the lower reinforcing ratio, and thereby have the higher restraint factor. Although there were limited available test results, the following can be tentatively suggested to apply to slabs with this form of restraint.

For $\frac{C}{B} \leqslant 6.0$ and $q \leqq 0.1$, $F_r \geqq 0.50$,

and

For $6.0 \leqslant \frac{C}{B} \leqslant 9.0$ and $q \leqq 0.2$, $F_r \geqq 0.25$

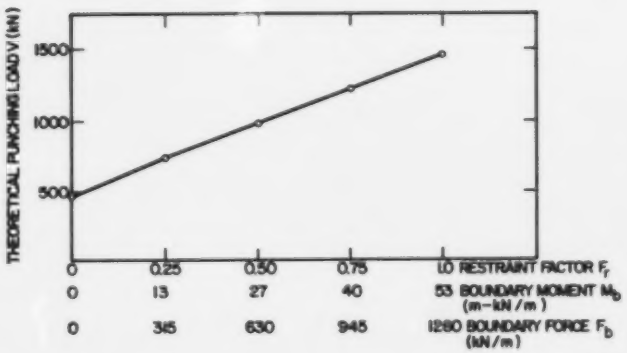


Figure 4/ Variation of Theoretical Punching Load with Restraint Factor

4/ Tests and Observations

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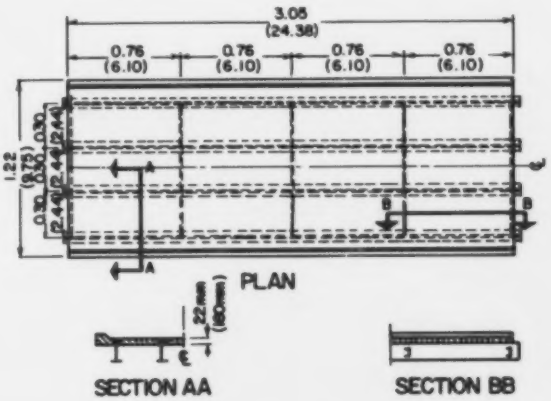
The behaviour of I-beam bridge slabs was investigated by testing a total of nine 1/8th scale direct models of 80 ft. (24.4 m) span 4-beam and 3-beam bridges. Apart from the usual difficulties of small scale modelling, special problems arose in the accurate modelling of all section properties, stud shear connector behaviour and dead load stresses appropriate to unshored construction. The solutions to these problems are described in detail elsewhere by Batchelor and Hewitt (1972) and Hewitt (1972). The 4-beam prototype and model bridges are detailed in Figure 5. A view of a 4-beam model bridge superstructure is given in Figure 6. The 3-beam model was similar to the 4-beam model except the beam spacing was 18 in. (460 mm) and the beams were supplemented with bottom flange plates.

The deck slabs of the bridges were tested to failure under single concentrated loads applied mid-way between adjacent bridge beams through a steel plate bearing on a neoprene pad. The contact area modelled an ellipse with major and minor axes of 30 in. (760 mm) and 20 in. (510 mm) respectively, which represents the assumed contact area of the pneumatic tires of large earth moving equipment. A view of a model bridge and the testing arrangement is given in Figure 7.



NOTE: ALL PROTOTYPE DIMENSIONS IN PARENTHESES

BEAMS 4 I 9.5(48WWF-R197)
DIAPHRAGMS 2 L33(18 42.7)

Figure 5/ Details of Prototype and Model Four-Girder Bridge

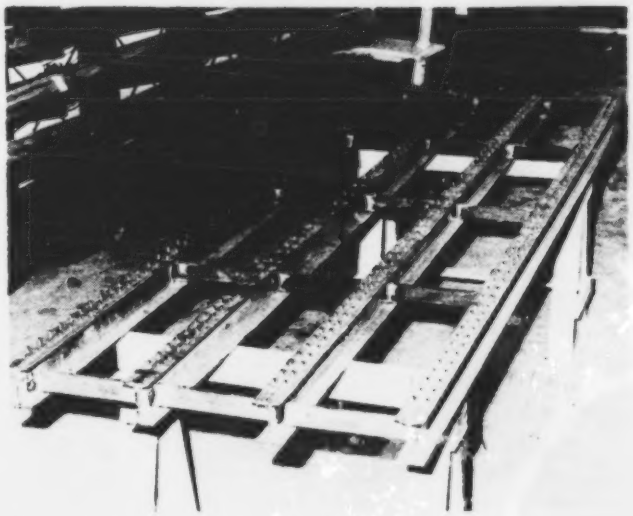


Figure 6/ View of Four-Beam Model

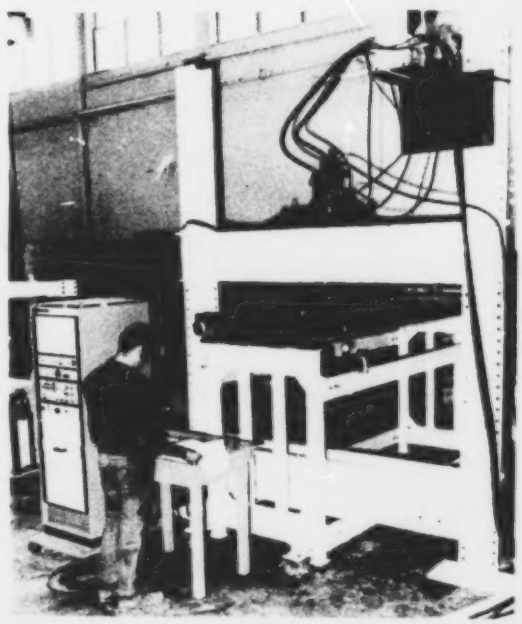


Figure 7/ View of Testing Arrangement

Deck slabs with orthotropic and with isotropic reinforcement, as well as plain concrete slabs, were tested. The orthotropic reinforcement modelled the reinforcement of conventional deck slabs. Slabs with isotropic reinforcement at mid-depth of the slab, and therefore with maximum cover, were also tested. The influences of slab span and thickness, load position, dead load stresses, reinforcement ratio, and concrete strength on the ultimate strength of the deck slabs were studied, together with the punching strength in a hogging moment region.

The model bridges tested are numbered 1 to 9. A bridge panel is defined as that portion of the deck bounded by adjacent bridge beams and adjacent diaphragms, and panels across a bridge are referred to collectively as strips. The bridges and the tests are described generally in Table 1. Bridges numbered 1 to 8 were 4-beam bridges, bridge 9 was a 3-beam bridge.

A total of 68 tests to failure were carried out. All but one of the reinforced panels and some of the unreinforced panels failed by punching. Failure by punching usually left a neat elliptical hole, a little larger than the loaded area, in the top of the slab and a pushed-through plug of concrete in the form of a frustum of a cone with an approximately circular base. The cracking patterns were similar for the slabs of the 4-beam and 3-beam bridges.

As the load was applied, visible cracking in the form of longitudinal and diagonal cracks was observed on the underside of the slab. This occurred directly beneath the loaded area and usually commenced at a load of between 25% and 50% of the failure load. Subsequently, transverse cracks appeared on the upperside of the slab on either side of the loaded area in the vicinity of the adjacent bridge beams. As failure approached, the underside cracking developed into a complete pattern of cracks radiating from beneath the loaded area, and the upperside longitudinal cracks lengthened and curved around the loaded area. Prior to failure, the cracking pattern indicated imminent flexural failure in an elliptical fan mechanism and never suggested failure by punching. Although accelerated creep usually gave some warning, failure was always explosive. Cracking rarely extended into adjacent panels and, generally, the higher the percentage of reinforcement, the more symmetrical and closely spaced was the cracking. Views of the cracking pattern of a 4-beam bridge panel with 0.2% isotropic reinforcement after failure by punching, are given in Figure 8.

Some unreinforced panels and one panel with a low percentage of reinforcement failed in flexure in elliptical fan mechanisms. Up to failure the cracking behaviour was similar to that for all other slabs but failure occurred after much creep and with some crushing of the concrete on the upper side of the slabs along lines radiating from the loaded area. The major axis of the fan mechanism was aligned with the major axis of the loaded area, and the cracking pattern was contained within adjacent bridge beams.

4.1/ Deflection Behaviour

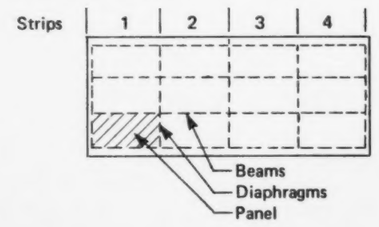
Figure 9 shows typical variations of net slab deflection with load for a 4-beam bridge. There was considerable scatter of the net slab deflections for panels of the same design but the tests plotted in Figure 9 were chosen to show the general trend of deflection behaviour. Panels with low percentages of reinforcement failed after relatively large deflections. Panels with 0.6% isotropic reinforcement behaved similarly to panels with orthotropic reinforcement and failed after only small deflections.

Table 1/ Models Constructed

Bridge number	Reinforcement* Strip 1**	Strip 2	Strip 3	Strip 4	Special Tests
1	0 RT	0 RT	0 RT	0 RT	
2	0 RT	0 RT	0 RT	0 RT	Full dead load compensation.
3	0 RT	0 RT	0 RT	0 RT	Some panels with dead load compensation.
4	0 RT	0 RT	0 RT	0 RT	Full dead load compensation.
5	0 RT	zero	0.4%	0.2%	
6	0.6%	0.2%	0.6%	M	Hogging moment in strips 2 and 3.
7	0.4%	0.2%	zero	0.6%	
8	0.4%	zero	M	0.2%	
9	0.4%	0.2%	zero	0.6%	

*Designation of reinforcement:
ORT indicates orthotropic reinforcement
0.6, 0.4, 0.2 } is percentage of isotropic reinforcement
M indicates mid-depth isotropic reinforcement

**Designation of strips:



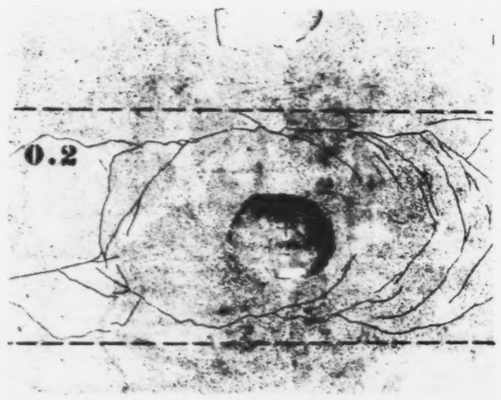


Figure 8a/ Top View of Cracking Pattern after Failure of Panel with 0.2% Reinforcement

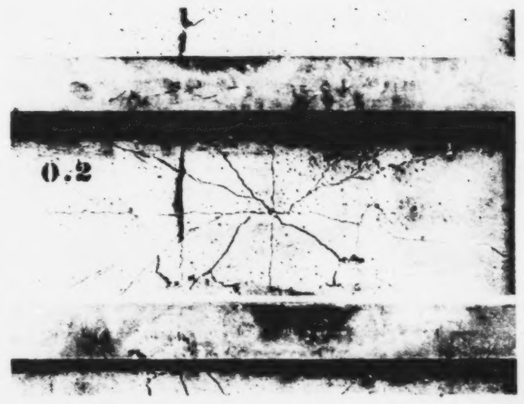


Figure 8b/ Bottom View of Cracking Pattern After Failure of Panel with 0.2% Reinforcement

Typical Output

```
*SELECTING BOUNDARY RESTRAINTS
==============================
DEFLECTION =                          1.090 INS
ULTIMATE PUNCHING LOAD PU =           49.59 KIPS
ALLOWING FOR DOWEL EFFECT 1.29PU =    59.51 KIPS

CONSIDERING BOUNDARY RESTRAINTS   (RESTRAINT FACTOR    0.25)
==============================================================
BOUNDARY FORCE      1781.7 LBS/IN
BOUNDARY MOMENT     1506.8 IN LBS/IN
DEFLECTION =                          0.510 INS
ULTIMATE PUNCHING LOAD PU =          124.03 KIPS   (INCREASE   150.1%)
ALLOWING FOR ORIGINAL DOWEL EFFECT PU =  103.95 KIPS   (INCREASE   125.1%)

CONSIDERING BOUNDARY RESTRAINTS   (RESTRAINT FACTOR    0.50)
==============================================================
BOUNDARY FORCE      3606.6 LBS/IN
BOUNDARY MOMENT     2960.6 IN LBS/IN
DEFLECTION =                          0.365 INS
ULTIMATE PUNCHING LOAD PU =          138.80 KIPS   (INCREASE   280.2%)
ALLOWING FOR ORIGINAL DOWEL EFFECT PU =  190.72 KIPS   (INCREASE   233.9%)

CONSIDERING BOUNDARY RESTRAINTS   (RESTRAINT FACTOR    0.75)
==============================================================
BOUNDARY FORCE      5410.0 LBS/IN
BOUNDARY MOMENT     4440.8 IN LBS/IN
DEFLECTION =                          0.292 INS
ULTIMATE PUNCHING LOAD PU =          250.30 KIPS   (INCREASE   404.8%)
ALLOWING FOR ORIGINAL DOWEL EFFECT PU =  260.30 KIPS   (INCREASE   337.4%)

CONSIDERING BOUNDARY RESTRAINTS   (RESTRAINT FACTOR    1.00)
==============================================================
BOUNDARY FORCE      7254.5 LBS/IN
BOUNDARY MOMENT     5869.5 IN LBS/IN
DEFLECTION =                          0.248 INS
ULTIMATE PUNCHING LOAD PU =          309.40 KIPS   (INCREASE   523.8%)
ALLOWING FOR ORIGINAL DOWEL EFFECT PU =  319.31 KIPS   (INCREASE   436.5%)

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DECKBFU

```
      COMMON/HEADER/TITLE(8),ECBND(,),MADEBY(3),WPND(3),DAYT(3),JOBNO(2)
      COMMON/ALTER/LINE
      COMMON /TSCCOM/ INTRIV
      DATA NUMN/'N='/
      LOGICAL TRACE,INTRIV
C
 1000 NSWIZL = 1
      KOLM2  = 10
      CALL HCARD(INOPRG,NICK,NSWIZL,LASTJP,KOLM2,KOLM3,KOLM4,KOLM5,
     1 KOLM10)
      IF(LASTJP .EQ. 999999) GO TO 999
      IF(INOPRG .NE. 1103) GO TO 998
      TRACE = .FALSE.
      IF(NICK .EQ. 1) TRACE = .TRUE.
C
      CALL BCFROM(TRACE)
C
  998 NSWIZL = 2
      CALL HCARD(INOPRG,NICK,NSWIZL,LASTJP,KOLM2,KOLM3,KOLM4,KOLM5,
     1 KOLM10)
      GO TO 1000
C
  999 STOP
      END
      SUBROUTINE BCFROM(TRACE)
C                    PUNCHING SHEAR WITH BOUNDARY RESTRAINTS
C                    (KINNUNEN, NYLANDER & MEWITT) PROGRAM
C
      COMMON/PRMADC/NARG,INDATA(20),NTYPE(20),NINT(20),REAL(20),
     1NTEXT(20)
      COMMON/ISOCOM/INTRIV
C
      LOGICAL INTRIV
      LOGICAL TRACE
      REAL MU,KY,AZ
      INTEGER TEST
      DIMENSION TITLE(12),TEST(2)
      DATA NUMN/'N='/
      KOUNT = 0
                  READ DATA
C
      IF(.NOT.INTRIV)GO TO 40
  400 D=0.
      M=0.
      B=0.
      C=0.
      SIGCY=0.
      MU=0.
      SIGSY=0.
      BM=0.
      BF=0.
      ES=0.
      WRITE(6,410)
  410 FORMAT(1X,'ENTER TEST NO.*)
      READ(5,900)INDATA
  900 FORMAT(20A4)
      CALL PREAD
      IF(NARG.EQ.0)GO TO 420
      TEST(1) = NTEXT(1)
      TEST(2) = NTEXT(2)
C
  420 WRITE(6,430)
  430 FORMAT(1X,'ENTER OVERALL THICKNESS OF SLAB (INS)*)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 440
      D=REAL(1)
C
  440 WRITE(6,450)
  450 FORMAT(1X,'ENTER EFFECTIVE THICKNESS OF SLAB (INS)*)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 460
      M=REAL(1)
C
  460 WRITE(6,470)
  470 FORMAT(1X,'ENTER EQUIVALENT DIAMETER OF LOADED AREA (INS)*)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 480
      B=REAL(1)
C
  480 WRITE(6,490)
  490 FORMAT(1X,'ENTER EQUIVALENT DIAMETER OF SLAB (INS)*)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 500
      C=REAL(1)
C
  500 WRITE(6,510)
  510 FORMAT(1X,'ENTER CYLINDER STRENGTH OF CONCRETE (PSI)*)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 520
      SIGCY=REAL(1)
C
  520 WRITE(6,530)
  530 FORMAT(1X,'ENTER RATIO OF REINFORCEMENT *)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 540
      MU=REAL(1)
  540 WRITE(6,550)
  550 FORMAT(1X,'ENTER YIELD POINT STRESS OF STEEL (PSI)*)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 560
      SIGSY=REAL(1)
  560 WRITE(6,570)
  570 FORMAT(1X,'ENTER BOUNDARY MOMENT (IN-LBS/IN)*)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 580
      BM=REAL(1)
```

```
C
  580 WRITE(6,590)
  590 FORMAT(1X,'ENTER BOUNDARY FORCE (LBS/IN)*)
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 595
      BF=REAL(1)
C
  595 WRITE(6,596)
  596 FORMAT(1X,'ENTER MODULUS OF ELASTICITY OF STEEL (PSI)*)
      READ(5,900)INDATA
      CALL PREAD
      ES=REAL(1)
      IF(NARG.EQ.0)GO TO 51
      GO TO 51
   40 CALL READJV(JJV,NUMN)
      IF(JJV.EQ.NUMN) GO TO 999
      READ(5,5017EST,D,M,B,C,SIGCY,MU,SIGSY,BM,BF,ES
   50 FORMAT(2A4,2X,F5.1,F5.2,2F7.2,F5.0,F5.4,F5.0,2F10.2,F10.0)
      IF(D.LT.0.0)GO TO 999
      IF(ES.EQ.0)ES=30000000.0
C
   51 IF(INTRIV) GO TO 61
      KOUNT = KOUNT + 1
      IF(KOUNT.EQ.1) GO TO 61
      WRITE(6,134)
  134 FORMAT(/343,'**********',/)
      LINE = 61
      CALL PAGE (LINE,LIMIT)
   61 WRITE(6,55)TEST
   55 FORMAT(/23,'1.TEST ',2A4/13,15('*'))
      WRITE(6,57)D
   57 FORMAT(2X,'2.OVERALL THICKNESS OF SLAB',T40,F10.3,' INS')
      WRITE(6,60)M
   60 FORMAT(2X,'3.EFFECTIVE THICKNESS OF SLAB',T40,F10.3,' INS')
      WRITE(6,70)B
   70 FORMAT(2X,'4.EQUIVALENT DIAMETER OF LOADED AREA',T40,F10.3,' INS')
      WRITE(6,80)C
   80 FORMAT(2X,'5.EQUIVALENT DIAMETER OF SLAB',T40,F10.3,' INS')
      WRITE(6,90)SIGCY
   90 FORMAT(2X,'6.CYLINDER STRENGTH OF CONCRETE',T40,F10.1,' PSI')
      SIGCU=SIGCY/10.75+0.000021*SIGCY)
      WRITE(6,100)MU
  100 FORMAT(2X,'7.RATIO OF REINFORCEMENT',T40,F10.4)
      WRITE(6,110)SIGSY
  110 FORMAT(2X,'8.YIELD POINT STRESS OF STEEL',T40,F10.1,' PSI')
      WRITE(6,120)BM
  120 FORMAT(2X,'9.BOUNDARY MOMENT',T40,F10.2,' IN-LBS/IN')
      WRITE(6,124)BF
  124 FORMAT(13X,'10.BOUNDARY FORCE',T40,F10.2,' LBS/IN')
      WRITE(6,126)ES
  126 FORMAT(1X,'11.MODULUS OF ELASTICITY OF STEEL',T40,F10.1,' PSI')
C
      IF(.NOT.INTRIV)GO TO 130
C
      WRITE(6,600)
  600 FORMAT(1X,'IS THE INPUT CORRECT ?')
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0.0)GO TO 130
  610 WRITE(6,620)
  620 FORMAT(1X,'ENTER REF.NO.AND CORRECTED VALUE. IF NO MORE CHANGES   0
     1PUSH RETURN')
      READ(5,900)INDATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 51
C
      K=NINT(1)
      GO TO(11,12,13,14,15,16,17,18,19,20,21),K
C
   11 TEST(1) = NTEXT(2)
      TEST(2) = NTEXT(3)
      GO TO 610
   12 D=REAL(2)
      GO TO 610
   13 M=REAL(2)
      GO TO 610
   14 B=REAL(2)
      GO TO 610
   15 C=REAL(2)
      GO TO 610
   16 SIGCY=REAL(2)
      GO TO 610
   17 MU=REAL(2)
      GO TO 610
   18 SIGSY=REAL(2)
      GO TO 610
   19 BM=REAL(2)
      GO TO 610
   20 BF=REAL(2)
      GO TO 610
   21 ES=REAL(2)
      GO TO 610
  130 BMP=12.*BM*BM/1000.0
      IFLAG=1
      X=0.0
      BMHOLD=BM
      BM=0.0
      BFHOLD=BF
      BF=0.0
      IF(MU.NE.0.0)GO TO 133
      PU=0.001
      GO TO 240
C
  133 KFLAG=1
      LFLAG=1
      MFLAG=1
      M=0
      M=0
      NY=0
      NSY=0
      NSX=0
      PDIFF=0.0
      XDIFF=0.0
      YDELTA=0.1
      XDELTA=0.01
      IF(IFLAG.EQ.1)GO TO 136
      BM=BMHOLD
      BF=BFHOLD
      IF(BM.EQ.0.0.AND.BF.EQ.0.0)GO TO 40
C               SELECT X
      X=1.0
      IF(BM.EQ.0.0)X=0.0
C               SELECT Y/M
  136 YONM=0.5
C               CALCULATE P1
  140 Y=YONM*M
      IF(M.LT.200.AND.M.LT.50)GO TO 160
      IF(IFLAG.EQ.1)WRITE(6,241)
      IF(IFLAG.EQ.2)WRITE(6,243)
      WRITE(6,150)
  150 FORMAT(/1X,' ***ERROR*** SOLUTIONS HAVE NOT CLOSED*/)
      IF(IFLAG.EQ.1)GO TO 246
      WRITE(6,244)BF
      WRITE(6,245)BM
      GO TO 246
  160 HOLD1=(1.0/4.7)*(1.0+Y/B)*ALOG(C/(B+2.0*Y))
      KY=(C-B)/12.0*(M-Y/3.0))
      AZ=KY-3*C/(4.0*(M-Y/3.0))
      HOLD2=AZ*HOLD1
      HOLD3=-AZ-1.0
      HOLD4=HOLD1*1.0
      HOLD5=(HOLD3**2-4.0*HOLD2*HOLD4)
      IF(HOLD5.GE.0.0)GO TO 168
      TANA=-HOLD3/(2.0*HOLD2)
      JFLAG=2
      GO TO 169
  168 TANA=(-HOLD3-SQRT(HOLD5))/(2.0*HOLD2)
      JFLAG=1
  169 FALFA=TANA*(1.0-TANA)/(1.0+TANA**2)
      BOM=B/M
      F=14.22
      IF(BOM+GT.2.0)GO TO 170
      SIGT=825.0*(0.35+SIGCU/(F+500.0))*(1.0-0.22*BOM)**5
      GO TO 180
  170 SIGT=460.0*(0.35+SIGCU/(F+500.0))**F
  180 P1=3.14*B*(BOM+YONM*(1+2.0*Y)*SIGT*FALFA)**2/(8*Y)
C                 CALCULATE P2  (ASSUMING TWO WAY REINFORCEMENT)
      IF(BOM+GT.2.0)GO TO 190
      PS1=0.0035*(1.0-0.22*BOM)*(1.0+0.5*B/Y)
      GO TO 200
  190 PS1=0.0019*(1.0+0.5*B/Y)
  200 RS=ME1*PS1*(1.0-YONM)/SIGSY
      C0=0.5*B+1.2*M
      HOLD6=MU*SIGSY*BOM
      IF(RS.LE.C0)GO TO 210
      IF(RS.GT.0.5*C.AND.RS<0.5*C
      R1=HOLD6*((RS-C0)+RS*ALOG(0.5*C/RS))
      AZ=HOLD6*C0
      GO TO 220
  210 R1=HOLD6*RS*ALOG(0.5*C/C0)
      AZ=HOLD6*RS
  220 DEFL=PSI*(C-B)/2.0
      P2=0.284*(R1*AZ*BF*C*(M-Y/3.0-DEFL)/(2.0*(M-Y/3.0)))/AZ
C                 COMPARE P1 & P2
      PBMGLD=PDIFF
      PDIFF=ABS(P1-P2)
      PERR1=100.0*PDIFF/P1
      IF(PERR1.LT.1.0)GO TO 230
      IF(MY.NE.0.AND.PDIFF.GT.PBMOLD)LFLAG=2
C                 ITERATION OF Y/M
      IF(KFLAG.NE.1)GO TO 223
      IF(LFLAG.EQ.2)GO TO 225
      IF(MY.GT.15)GO TO 225
      YHOLD=YONM
      YONM=YONM*0.5*(1+P2/P1)
      IF(MY.GT.10)YONM=(YONM+YHOLD)/2.0
      M=M+1
      MY=MY+1
      GO TO 140
C               SEARCH FLR Y/M
  225 KFLAG=2
  226 YONM=YONM+YDELTA
      M=M+1
      NSY=NSY+1
      GO TO 140
  227 IF(PDIFF.LT.PBMOLD)GO TO 226
      YONM=YONM-YDELTA
      PDIFF=PBMOLD
      IF(KFLAG.EQ.3.OR.NSY.GT.1)GO TO 228
      KFLAG=3
      YDELTA=-YDELTA
      GO TO 226
  228 KFLAG=2
      YDELTA=0.1*YDELTA
      NSY=0
      GO TO 226
  230 PU=(P1+P2)/2000.0
      IF(KFLAG.EQ.3)GO TO 240
      XCHECK=BMP/PU
C               COMPARE X & XC=ECK
      XBMGLD=XDIFF
      XDIFF=X-XCHECK
      PERR2=ABS(100.0*XDIFF/X)
      IF(PERR2.LT.1.0)GO TO 240
C                 ITERATION OF X
      YDELTA=0.1
      MY=0
      NSY=0
      KFLAG=1
      LFLAG=1
      IF(MFLAG.NE.1)GO TO 237
      IF(M.NE.0.AND.XDIFF.GT.XBMOLD)GO TO 235
      X=(X+XCHECK)/2.0
      M=M+1
      GO TO 140
C               SEARCH FOR X
  235 MFLAG=2
  236 X=X+XDELTA
      M=M+1
      NSX=NSX+1
      GO TO 140
```

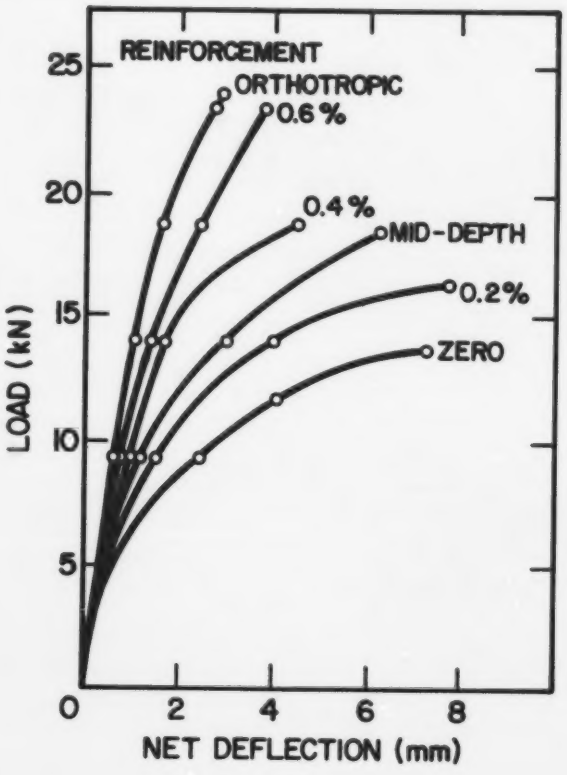

Figure 9/ Typical Variations of Net Slab Deflection with Load for Four-Beam Bridges

4.2/ Punching Strength

4.2.1/ Slabs with Orthotropic Reinforcement

There was considerable variation of the failure loads for slabs of the same design. A statistical analysis of all the results of tests of orthotropically reinforced panels indicated that the variation of failure load could be attributed to unavoidable variations of the slab dimensions. The analysis showed that the failure load is not significantly influenced by the position of the panel, by previous failures in adjacent panels, and by the strength of the concrete or dead load stresses.

Figure 10 is a plot of test punching load for orthotropically reinforced panels versus effective depth to the bottom transverse reinforcement (d_{bt}) and shows the regression line determined using all results. There was no significant difference between this regression line and those determined using only the results of tests with and without dead load compensation. It should not be inferred that the regression line can necessarily be extrapolated for slabs of properties outside the range of those which have been included in the analysis. The scale of multiples of design wheel load, impact not included, shows that the factor of safety against failure by punching was never less than 17.

The punching load for a slab of design dimensions determined using the regression line is 5.21 kips (23.2 kN). If a design wheel load of 16 kips (71 kN) is assumed, the factor of safety against punching is approximately 21. If a design wheel load of 16 kips (71 kN) and an impact factor of 0.3 are assumed, the factor of safety is approximately 16. The bridge deck design is obviously very conservative.

In a few cases the load was applied at four points simultaneously to simulate a truck with wheels 6 ft. (1.8 m) apart on axles 14 ft. (4.2 m) apart. By measuring the extreme bottom fibre strains of the bridge beams it was shown that for a conventional I-beam bridge under truck loading, beam failure is to be expected well before slab failure. This is further indication that the bridge deck slab design is conservative.

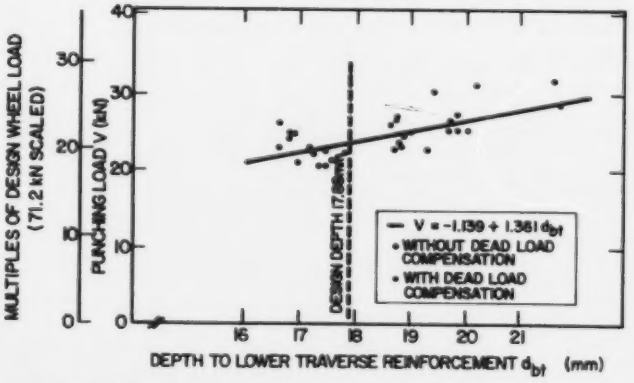

Figure 10/ Punching Load vs Depth to Reinforcement – Orthotropic Reinforcement

4.2.2/ Slabs with Isotropic Reinforcement

The average test punching loads for each nominal percentage of reinforcement and for panels of both 4-beam and 3-beam bridges are shown plotted against average reinforcement percentage in Figure 11. The average reinforcement percentages were calculated using the means of all the measured effective depths of panels with the respective nominal reinforcement. Both flexural and punching failures occurred in 4-beam bridge panels with 0.2% and with zero reinforcement, and it was assumed that flexural and punching strengths were the same for these panels. Only flexural failures,

at an average load of 2.12 kips (9.43 kN), occurred in the unreinforced panels of the 3-beam bridge and results of the tests of these panels were excluded from the analysis.

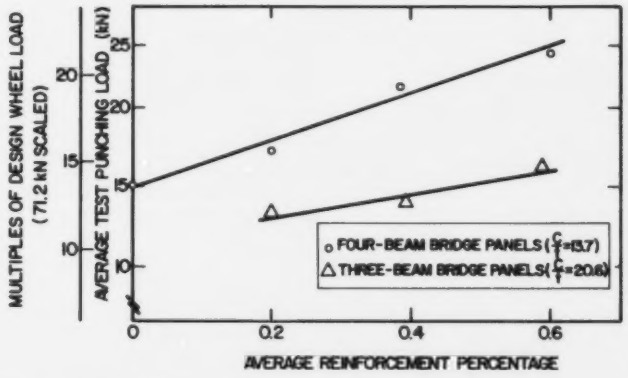

Figure 11/ Variation of Test Punching Load with Reinforcement Percentage — Isotropic Reinforcement

Figure 11 shows that the punching strength increases with increases of reinforcement and with decrease of span to thickness ratio (C/t). The scale of multiples of design wheel loads indicates that the factor of safety is always high, even when the span to depth ratio is unusually large, and that the factor of safety for a panel of a bridge of conventional dimensions would be approximately 13, if impact is ignored. There was some scatter of the test results but the factor of safety for unreinforced panels of 4-beam bridges was never less than 10.

In an elastic design only the strength due to reinforcement is considered. Only in an ultimate strength design can both the strength due to the reinforcement and the very significant additive inherent strength of the unreinforced panel be utilized.

In tests on a model bridge in which hogging moment was induced, it was shown that such a moment has little, if any, effect on the punching load. It became apparent that extensive transverse cracking can be tolerated before any decrease in the punching strength of the bridge deck is to be anticipated.

The restraint factors for panels of the 4-beam and 3-beam bridges are given in Table 2. The theory was not considered to be applicable to the unreinforced panels of the 3-beam bridges, which all failed in flexure, or to the panels with mid-depth reinforcement. The restraint factors quantify the actual boundary restraints of the panels in terms of the idealized maximum restraints on the boundaries of the idealized equivalent slabs. The restraint factors generally increase with increase of percentage reinforcement. This is contrary to the findings from the analysis of the tests by Taylor and Hayes (1970) and Aoki and Seki (1975) discussed previously, and is thought to be due largely to the fact that in bridge panels the top reinforcement may also contribute to the boundary restraint.

However, noting the reinforcing indices q and the diameter ratios C/B given in Table 2, it is seen that the limits of the restraint factor, which have been tentatively proposed, also apply here although the increase of restraint factor with increase of reinforcement is not utilized. With the high factors of safety, slabs with very low percentages of reinforcement are of interest, and for design purposes a restraint factor of 0.50 is proposed for use in the calculation of the punching strength of the deck slabs of composite I-beam bridges. This would be satisfactory for a slab with a span to thickness ratio as high as 20 and with a reinforcement percentage as low as 0.2%.

Table 2/ Restraint Factors for Panels of Four-Beam and Three-Beam Bridges

Isotropic Reinforcement (%)	Four-Beam Bridge Slabs C/B = 3.7				Three-Beam Bridge Slabs C/B = 3.6			
	Number of Tests	q (average)	V_{test} (average) (kN)	Restraint Factor	Number of Tests	q (average)	V_{test} (average) (kN)	Restraint Factor
0.6	7	0.055	23.66	0.74	2	0.054	16.46	0.59
0.4	5	0.036	21.57	0.66	2	0.036	14.14	0.52
0.2	7	0.018	18.01	0.60	2	0.018	13.48	0.55
zero	3	0	16.50	0.57	2[a]	0	—	—

Note: [a]) Both slabs failed in flexure
1 kN = 0.225 kip

5/Design Recommendations

5/ Design Recommendations

In accordance with the philosophy of the ACI Code, the ultimate strength of a structure is expected to exceed about two and a half times the design live load. The factor of safety against failure by punching of an unreinforced panel of a 4-beam bridge, assuming a wheel load of 16 kips (71 kN) and an impact factor of 0.3, is approximately 10. The study indicated that cracking under working loads is not a problem and that slab failure is highly unlikely, particularly since the bridge beams would probably fail first under truck loading.

Clearly, if only the punching strength is to be considered, reinforcement is theoretically not required in the deck slab of I-beam bridges. However, the AASHTO Standard Specifications for Highway Bridges (1973) recommends that

> "not less than 0.125 in.^{2} of reinforcement per foot (264 mm^2/m) shall be placed in each direction of all concrete surfaces to resist the formation of temperature and shrinkage cracks".

This amounts to approximately 0.2% reinforcement for a 7.0 in. (178 mm) slab with a 1.5 in. (38 mm) cover. The authors recommend this to be the maximum reinforcement required for the deck slabs of composite I-beam bridges. Using 0.2% isotropic reinforcement rather than the conventional orthotropic reinforcement as in the prototype bridge, the reinforcement requirement is reduced by about 66%.

The tests of panels with 0.2% reinforcement are therefore of particular interest. The test punching loads of the 4-beam bridge panel with 0.2% reinforcement are plotted against the measured effective depths to the bottom transverse reinforcement in Figure 12. The line of regression of the punching load on the effective depth is shown. The scale of multiples of design wheel load shows that a slab of design dimensions has a factor of safety of approximately 12. Considering a wheel load of 16 kips (71.2 kN) and an impact factor of 0.3, the factor of safety is greater than 9.

Thus the design of the deck slab of I-beam bridges can be made very simple because only the AASHTO requirements regarding the temperature and shrinkage reinforcement need be satisfied. Assuming a restraint factor of 0.50, the ultimate strength of any slab with a span to depth ratio within the range of 14 to 21 of the tested panels can be estimated. Figure 13 shows a design curve, derived using the outlined theory, which relates punching loads and span to depth ratios of panels with 0.2% reinforcement. The punching loads were calculated assuming slabs of the following dimensions and properties:

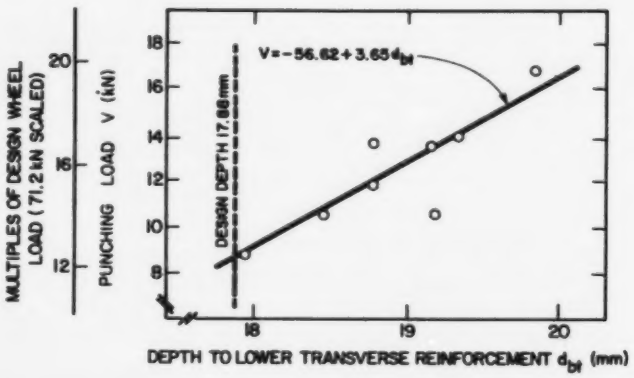
MULTIPLES OF DESIGN WHEEL LOAD (71.2 kN SCALED)
PUNCHING LOAD V (kN)
V=-56.62+3.65d_bt
DESIGN DEPTH 17.88mm
DEPTH TO LOWER TRANSVERSE REINFORCEMENT d_bt (mm)

Figure 12/ Punching Load vs Depth to Reinforcement – 0.2% Isotropic Reinforcement

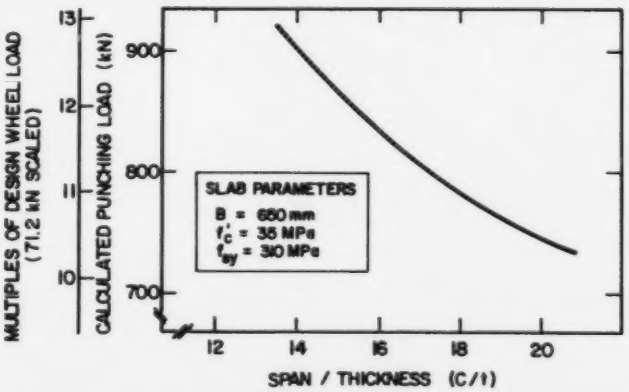
MULTIPLES OF DESIGN WHEEL LOAD (71.2 kN SCALED)
CALCULATED PUNCHING LOAD (kN)
SLAB PARAMETERS
B = 650 mm
f'c = 35 MPa
fsy = 310 MPa
SPAN / THICKNESS (C/t)

Figure 13/ Variation of Calculated Punching Load with Span/Thickness Ratio Assuming a Restraint Factor of 0.5 – 0.2% Isotropic Reinforcement

Slab thickness	7 in. (178 mm)
Slab effective depth	5 in. (127 mm)
Diameter of loaded area	25.6 in. (650 mm)
Slab diameter	96 to 144 in. (2.44 to 3.66 m)
Reinforcement	0.2%
Yield stress of reinforcement	45 ksi (310 MPa)
Compressive strength of concrete	5000 psi (34 MPa)
Restraint factor	0.50

The loaded area was assumed to be circular and of the same perimeter as the assumed elliptical tire contact area. The slab diameter was taken as equal to the bridge beam spacing. The design curve could be used to check the ultimate strength and the factor of safety against failure by punching of any deck slab of comparable dimensions and with the required reinforcement of 0.2%.

The preceding discussion does not apply to the exterior cantilevering portions of I-beam bridge deck slabs and can only apply to panels with adequate boundary restraint, which is ensured by the shear connectors, bridge beams, diaphragms, and by the continuity of the slab itself. According to the AASHTO Specifications, "the maximum pitch of shear connectors shall not exceed 24 in. (610 mm), except over the interior supports of continuous beams where wider spacings may be used to avoid placing connectors at locations of high stresses in the tension flange". The maximum spacing of the shear connectors in the model bridges was 4 in. (102 mm). For a prototype bridge, shear connectors at spacings of up to 32 in. (810 mm) could theoretically provide adequate restraint and some reduction in the factor of safety for the bridge deck could result from shear connector spacings in excess of 32 in. (810 mm). Diaphragms may need to be provided to prevent spreading of the bridge beams due to in-plane forces in the loaded deck slab, and it appears that the AASHTO requirements are adequate. The end edges of the deck slab should always be stiffened.

Mid-depth reinforcement offers the advantage of ensuring maximum reinforcement cover but it does not satisfy the AASHTO requirements regarding temperature and shrinkage reinforcement. If, despite this, mid-depth reinforcement is used to resist the formation of temperature and shrinkage cracks, the ultimate strength of the slab could be conservatively estimated by assuming it to be unreinforced.

6/Conclusions and Recommendations

6/ Conclusions and Recommendations

The mechanical model used by Kinnunen and Nylander (1960) can be modified to predict the punching strength of restrained slabs. This factor can be empirically determined for slabs of unknown boundary restraint.

The current method of design of the deck slab of a steel/concrete composite bridge results in the wasteful use of reinforcement. The factor of safety against failure by punching under a single wheel load can be expected to be approximately 16. Under truck loading, beam failure can be expected before slab failure. The punching strengths of such slabs vary inversely with span, and directly with reinforcement ratio and effective depth.

The punching strength is not influenced significantly by the following factors:

1/ The position of the load on the bridge deck slab.
2/ Previous failures of the slab in adjacent panels.
3/ A hogging moment.
4/ Dead load stresses and deflections.
5/ The strength of the concrete of the slab.

It has been shown that, theoretically no reinforcement is required in the deck slabs of composite I-beam bridges if only the ultimate strength of the designed structure is considered. Considering the AASHTO (1969) requirements regarding temperature and shrinkage reinforcement, 0.2% isotropic top and bottom reinforcement is recommended as the maximum requirement. This amounts to a reduction of 66% of the current reinforcement requirements. For a deck of conventional dimensions with 0.2% reinforcement, the factor of safety against failure by punching can be expected to be about 9.

A restraint factor of 0.50 should be used for the calculation of the punching strength for design purposes.

7/References

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Appendix A/Equations for Restrained Slabs

Appendix A

The following derivation for restrained slabs is based on the Kinnunen and Nylander model for simply supported slabs. By considering equilibrium of the slab in the vertical direction, the load at punching P, neglecting dowel effects is:

$$P = T \cdot \sin\alpha = \pi\left(\frac{B}{d}\right)\left(\frac{y}{d}\right)\frac{B+2y}{B+y}\, f_t\, f(\alpha)\, d^2 \quad (1)$$

where:

f_t = the stress in the imaginary shell

$$f(\alpha) = \frac{\tan\alpha\,(1-\tan\alpha)}{1+\tan^2\alpha}$$

The following expressions for f_t were established

for $\frac{B}{d} \leq 2$:

$$f_t = 825\left[0.35 + 0.3\left(\frac{f_{cube}}{150}\right)\right]\left[1 - 0.22\left(\frac{B}{d}\right)\right] \quad (2)$$

for $\frac{B}{d} < 2$:

$$f_t = 460\left[0.35 + 0.3\left(\frac{f_{cube}}{150}\right)\right] \quad (3)$$

where f_{cube} is the 6-inch cube strength of the concrete at 28 days.

Taking moments about the point of intersection of the resultant of the forces R_1 and the external load $P\beta/2\pi$ gives:

$$\frac{2\pi R_3}{P} = Ky - \cot\alpha - \left[\pi\left(\frac{M_b}{P}\right)\left(\frac{3c}{3d-y}\right)\right] \quad (4)$$

where:

$$Ky = \frac{3\,(C-B)}{2\,(3d-y)}$$

For convenience M_b/P is replaced with $X/4\pi$ where X is a factor of the order of unity (X is of the order of 1 for the conical collapse model). Thus:

$$X = 4\pi\left(\frac{M_b}{P}\right) \quad (5)$$

Now:

$$R_3 = \left(\frac{d^2}{2}\right)\left(\frac{y^2}{d}\right)\left(\frac{B}{2y}+1\right) f_{ct} \ln\left(\frac{C}{B+2y}\right) \quad (6)$$

where f_{ct} = the concrete stress in the tangential direction on the top surface of the slab at a radius of $B/(2+y)$. It was found that by using $f_t/f_{ct} = 2.35$ good agreement with the test results was achieved.

Using the expressions for P and R_3 given in equations 1 and 5, respectively, the following equation for the determination of α can be derived:

$$(K_z \tan\alpha - 1)\,\frac{1-\tan\alpha}{1+\tan^2\alpha} = \frac{1}{4.7}\left(1+\frac{y}{B}\right)\ln\left(\frac{C}{B+2y}\right) \quad (7)$$

where $K_z = K_y - \dfrac{3XC}{4\,(3d-y)}$ (8)

when $M_b = 0$, $X = 0$, and $K_z = K_y$, equation 8 is identical to that for simply supported slabs.

Taking moments about the point of intersection of $T\beta/2\pi$ in the conical shell and the resultant of the forces R_3, then

$$P = \frac{2\pi}{K_z}\left[R_1 + \left(\frac{R_2}{\beta}\right) + F_b\left(\frac{C}{2}\right)\right] \quad (9)$$

Again when $F_b = 0$, $M_b = 0$, $X = 0$ and $K_z = K_y$ equation 9 is identical to that for simply supported slabs.

Kinnunen and Nylander have shown that the tension reinforcement within a zone of radius r_s has yielded at punching failures in which

$$r_s = \frac{Es}{fsy}\psi\,(d-y)$$

where ψ = angle of rotation of the slab beyond the shear crack. Outside this zone the reinforcement is still within the elastic range. The term ψ was calculated using the following equations.

for $B/d \leq 2$

$$\psi = 0.0035\left[1 - 0.22\left(\frac{B}{D}\right)\right]\left(1+\frac{B}{2y}\right) \quad (10)$$

for $B/d > 2$

$$\psi = 0.0019\left(1+\frac{B}{2y}\right)$$

with $C_o = (B/2 + 1.8d)$ the following expressions can be derived for R_1 and R_2.

for $r_s > C_o$

$$R_1 = pf_{sy}d\left[(r_s - C_o) + r_s \ln\left(\frac{C}{2r_s}\right)\right]$$

and $R_2 = pf_{sy}\, dC_o\, \beta$

for $r_s \leq C_o$

$$R_1 = pf_{sy}\, dr_s \left(\frac{C}{2C_o}\right)$$

and $R_2 = pf_{sy}\, dr_s\, \beta$

The expression for C_o was established empirically for slabs with two-way reinforcement as:

$$C_o = \left(\frac{B}{2} + 1.8d\right)$$

Appendix B/ Computer Programs

Appendix B

Two computer programs were developed to evaluate the ultimate load capacity of restrained slabs. The programs are intended for use with time sharing terminals, thereby allowing the user to communicate with the system in a conversational mode. The computer printout can be immediately evaluated and, if necessary, the input variables can be modified to determine the effect of modifications on the ultimate capacity.

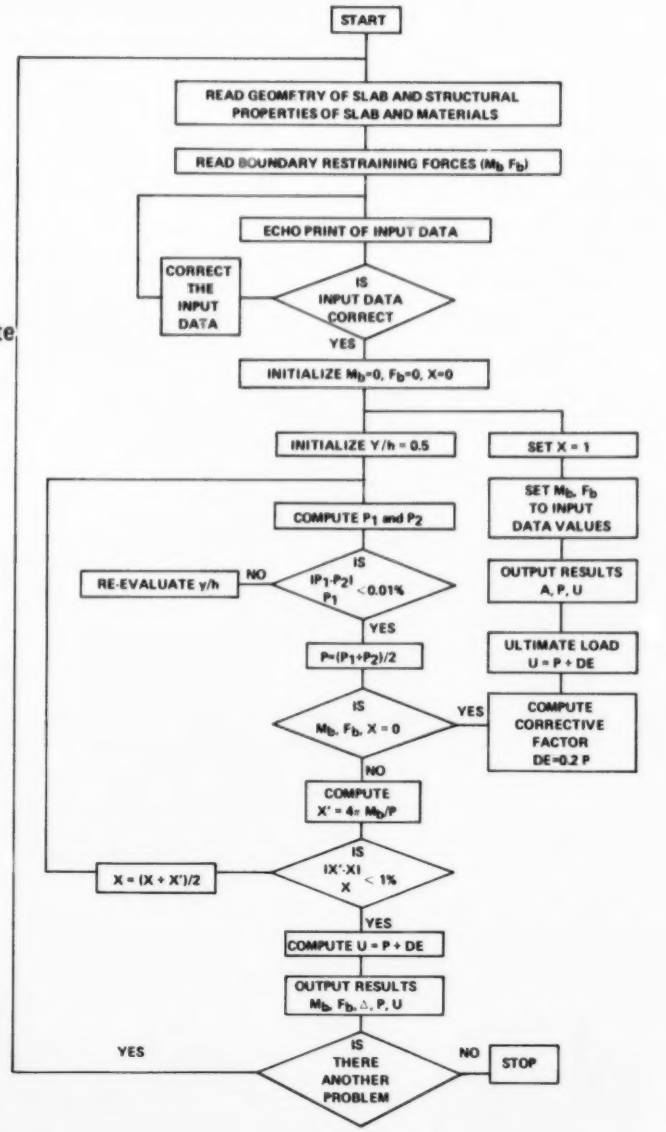
The program DECKBFK computes the capacity of a slab with known boundary restraint forces, both in-plane and bending. The output consists of the ultimate punching capacity and deflection for the slab for the case where no restraint forces are present, and for the case where the values of restraint forces are as specified.

The program DECKBFU computes the capacity of a slab with unknown boundary restraint forces. The slab capacity is calculated for the case of zero boundary restraint force ($F_r = 0$), for the case of maximum boundary restraint ($F_r = 1$), and for intermediate boundary restraint forces of 0.25 F_r, 0.50 F_r and 0.75 F_r. The output consists of the slab deflection, ultimate punching load and boundary force, and moment values for boundary restraint values (F_r) of 0.0, 0.25, 0.50, 0.75, 1.00.

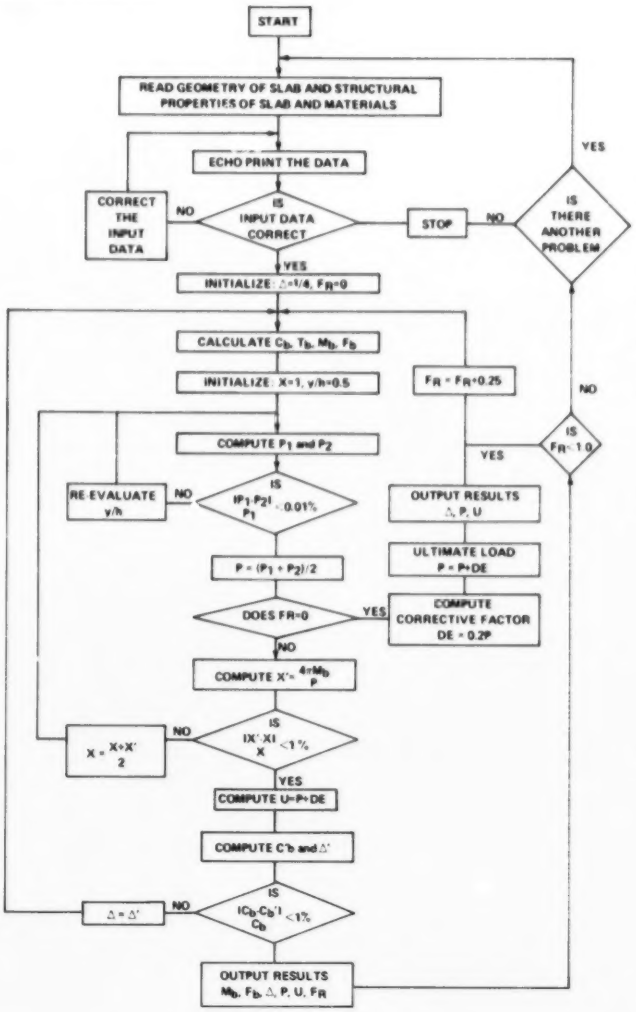
The format of the two programs is as uniform as possible and self-explanatory. If the loaded area is not circular, then the equivalent diameter of loaded area is taken as the diameter of a circle with the same perimeter as the loaded area. If the slab is not circular, the equivalent diameter of slab is taken as the diameter of the largest circle which could be inscribed within the area of the unsupported slab. For example, the equivalent diameter of slab for a composite concrete deck-steel girder bridge is taken as the distance between adjacent steel girders.

A flowchart and program listing for both programs are included.

CHART FOR DECKBFK



FLOWCHART FOR DECKBFU



DECKBFU

```
      COMMON/HEADER/TITLE(20),ECHO(12),MADE0Y(3),WPN0(3),DAYT(3),JOBNO(2)
      COMMON/ALINE/LINE
      COMMON /TSJCOM/ INTRIV
      DATA NUMB/'H'/
      LOGICAL TRACE,INTRIV
C
 1000 NSWIZL = 1
      KOL92  = 13
      CALL HCARD(NUPRG,NICK,NSWIZL,LASTJ0,KOL92,KOL93,KOL94,KOL99,
     1 KOL9113)
      IF(LASTJ0 .EQ. 999999) GO TO 999
      IF(NUPRG .NE. 1000) GO TO 999
      TRACE = .FALSE.
      IF(NICK .EQ. 13 TRACE = .TRUE.
C
      CALL PUNCHLETRACE(
C
  999 NSWIZL = 2
      CALL HCARD(NUPRG,NICK,NSWIZL,LASTJ0,KOL92,KOL93,KOL94,KOL99,
     1 KOL9113)
      GO TO 1000
C
  999 STOP
      END
      SUBROUTINE PUNCHLETRACE(
C                 PRINT HEAD
C                 FOR THE ANALYSIS OF HESTAIRED SLABS
      COMMON/PRFBOL/ NAPOS, INDATA(20), NTYPE(20), NINTE(20), REAL(20),   J
     1     NTEAT(20)
      COMMON /TSJCOM/ INTRIV
C
      LOGICAL INTRIV
      LOGICAL TRACE
      REAL MU,KV,KZ
      INTEGER TEST
      DIMENSION TITLE(20),TESTE(2)
      DATA NUMB/'H'/
      KOUNT = 0
C
C                 READ DATA
C
      IF(.NOT.INTRIV)GO TO 40
  430 I=2.
      M=2.
      N=2.
      C=2.
      SIGCY=2.
      MU=2.
      SIGSY=2.
      WRITE(6,431)
  431 FORMAT(1X,'ENTER TEST NO.')
      READ(5,900)INDATA
  910 FORMAT(20A4)
      CALL PREAD
      IF(NARG.EQ.0)GO TO 420
      TESTE(1) = NTEAT(1)
      TESTE(2) = NTEAT(2)
      TEST(2) = NTEAT(2)
C
  420 WRITE(6,430)
  430 FORMAT(1X,'ENTER OVERALL THICKNESS OF SLAB (INS)')
      READ(5,900)INDATA
      CALL PREAD
```

```
      IF(NARG.EQ.0)GO TO 420
      D=REAL(1)
C
  440 WRITE(6,450)
  450 FORMAT(1X,'ENTER EFFECTIVE THICKNESS OF SLAB (INS)')
      READ(5,900)INDATA
      CALL FREAD
      IF(NARG.EQ.0)GO TO 460
      H=REAL(1)
C
  460 WRITE(6,470)
  470 FORMAT(1X,'ENTER EQUIVALENT DIAMETER OF LOADED AREA (INS)')
      READ(5,900)INDATA
      CALL FREAD
      IF(NARG.EQ.0)GO TO 480
      B=REAL(1)
C
  480 WRITE(6,490)
  490 FORMAT(1X,'ENTER EQUIVALENT DIAMETER OF SLAB (INS)')
      READ(5,900)INDATA
      CALL FREAD
      IF(NARG.EQ.0)GO TO 500
      C=REAL(1)
C
  500 WRITE(6,510)
  510 FORMAT(1X,'ENTER CYLINDER STRENGTH OF CONCRETE (PSI)')
      READ(5,900)INDATA
      CALL FREAD
      IF(NARG.EQ.0)GO TO 520
      SIGCY=REAL(1)
C
  520 WRITE(6,530)
  530 FORMAT(1X,'ENTER RATIO OF REINFORCEMENT ')
      READ(5,900)INDATA
      CALL FREAD
      IF(NARG.EQ.0)GO TO 540
      MU=REAL(1)
C
  540 WRITE(6,550)
  550 FORMAT(1X,'ENTER YIELD POINT STRESS OF STEEL (PSI)')
      READ(5,900)INDATA
      CALL FREAD
      IF(NARG.EQ.0)GO TO 560
      SIGSY=REAL(1)
C
  560 WRITE(6,570)
  570 FORMAT(1X,'ENTER MODULUS OF ELASTICITY OF STEEL (PSI)')
C
      READ(5,900) INDATA
      CALL FREAD
      ES=30000000.0
      IF(NARG .EQ. 0) GO TO 51
      ES = REAL(1)
      GO TO 51
C
   40 CALL READJV(JJV,NUMH)
      IF(JJV .EQ. NUMH) GO TO 390
      READ(5,50)TEST,D,H,B,C,SIGCY,MU,SIGSY,ES
   50 FORMAT(2A4,2X,F5.1,F5.2,2F7.2,F5.0,F5.0,F5.0,2UA,F10.0)
      IF(D.LT.0.01)GO TO 390
      IF(ES.EQ.0)ES=30000000.0
C
   51 IF(INTRIV) GO TO 61
      LINE = 61
      IF(KOUNT.EQ.0) GO TO 61
      CALL PAGE (LINE,LIMIT)


   61 WRITE(6,55)TEST
   55 FORMAT(/1X,'1.TEST ',2A4/1A,15('='))
      WRITE(6,570)
   57 FORMAT(1X,'2.OVERALL THICKNESS OF SLAB',T40,F10.3,' INS')
      WRITE(6,60)H
   60 FORMAT(1X,'3.EFFECTIVE THICKNESS OF SLAB',T40,F10.3,' INS')
      WRITE(6,700B
   70 FORMAT(1X,'4.EQUIVALENT DIAMETER OF LOADED AREA',T40,F10.3,' INS')
      WRITE(6,80)C
   80 FORMAT(1X,'5.EQUIVALENT DIAMETER OF SLAB',T40,F10.3,' INS')
      WRITE(6,90)SIGCY
   90 FORMAT(1X,'6.CYLINDER STRENGTH OF CONCRETE',T40,F10.1,' PSI')
      SIGCU=SIGCY/(0.7+0.00025*SIGCY)
      WRITE(6,100)MU
  100 FORMAT(1X,'7.RATIO OF REINFORCEMENT',T40,F10.4)
      WRITE(6,110)SIGSY
  110 FORMAT(1X,'8.YIELD POINT STRESS OF STEEL',T40,F10.1,' PSI')
      WRITE(6,120)ES
  120 FORMAT(1X,'9.MODULUS OF ELASTICITY OF STEEL',T40,F10.1,' PSI'/)
C
      IF(.NOT.INTRIV)GO TO 130
C
      WRITE(6,600)
  600 FORMAT(1X,'IS THE INPUT CORRECT ?')
      READ(5,900)INDATA
      CALL FREAD
      IF(NARG.EQ.0)GO TO 51
  610 WRITE(6,620)
  620 FORMAT(1X,'ENTER KEY,NO.AND CORRECTED VALUE, IF NO MORE CHANGES  0
     1PUSH RETURN')
      READ(5,900)INDATA
      CALL FREAD
      IF(NARG.EQ.0)GO TO 51
C
      K=NENT(1)
      GO TO(11,12,13,14,15,16,17,18,19),K
C
   11 TEST(1) = NTEXT(2)
      TEST(2) = NTEXT(3)
      GO TO 610
   12 D=REAL(2)
      GO TO 610
   13 H=REAL(2)
      GO TO 610
   14 B=REAL(2)
      GO TO 610
   15 C=REAL(2)
      GO TO 610
   16 SIGCY=REAL(2)
      GO TO 610
   17 MU=REAL(2)
      GO TO 610
   18 SIGSY=REAL(2)
      GO TO 610
   19 ES=REAL(2)
      GO TO 610
C
  130 FF=0.0
      AF=0.0
      IFLAG=1
      X=0.0
      IF(MU.NE.0.0)GO TO 133
      MU=0.001
      DEFL=D/4.0
      GO TO 240
  133 KFLAG=1

      LFLAG=1
      YFLAG=1
      Y=0
      N=0
      I=0
      NY=0
      NSY=0
      NSX=0
      MDIFF=0.0
      XDIFF=0.0
      YDELTA=0.1
      XDELTA=0.01
      IF(IFLAG.EQ.1)GO TO 136
      FF=FF+0.25
      IF(FF.LE.1.0)GO TO 135
      WRITE(6,134)
  134 FORMAT(/30X,'**********FAILURE**********',/)
      KOUNT = KOUNT + 1
      IF(INTRIV) GO TO 390
      GO TO 40
C                     CALCULATE BOUNDARY RESTRAINTS
  135 TEN=FF*SIGSY*H**2
      CON=FF*((C/2.0)-(DEFL/4.0))*SIGCY*1.7/3.0
      BF=CON-TEN
      BM=TEN*(2.0*H-D)-CON*(H-D*13.0/16.0-DEFL*3.0/32.0)
      AMPF=12.566*BM/1000.0
      IF(I.GE.1)GO TO 140
C                     SELECT X
      X=1.0
C                     SELECT Y/H
  136 YONH=0.5
C                     CALCULATE P1
  140 Y=YONH*H
      IF(Y.LT.200.AND.Y.LT.50.AND.I.LT.5)GO TO 160
      IF(IFLAG.EQ.1)WRITE(6,241)
      IF(IFLAG.EQ.2)WRITE(6,245)FF
      WRITE(6,150)
  150 FORMAT(1A,' ***FAILURE*** SOLUTIONS HAVE NOT CLOSED*/)
      IF(IFLAG.EQ.1)GO TO 246
      WRITE(6,244)FF
      WRITE(6,2458)M
      GO TO 246
  160 HOLD1=(1.0/0.7)*(1.0+1/3)*ALOG(C/(0.2*Y))
      XY=(C-B)/(2.0*(H-Y/3.0))
      XZ=XY-XMC/(4.0*(H-Y/3.0))
      HOLD2=XZ+HOLD1
      HOLD3=-XZ-1.0
      HOLD4=HOLD1+1.0
      HOLD5=(HOLD3**2-4.0*HOLD2*HOLD4)
      IF(HOLD5.GE.0.0)GO TO 168
      TANA=-HOLD3/(2.0*HOLD2)
      JFLAG=2
      GO TO 169
  168 TANA=(-HOLD3-SQRT(HOLD5))/(2.0*HOLD2)
      JFLAG=1
  169 ALFA=TANA*(1.0-TANA)/(1.0+TANA**2)
      AONH=AF/H
      F=14.22
      IF(AONH.GT.2.03)GO TO 170
      SIGT=825.0*(0.33+SIGCU/(F*500.0))*(1.0-0.22*AONH)*F
      GO TO 180
  170 SIGT=460.0*(0.33+SIGCU/(F*500.0))*F
  180 P1=3.142*AONH*YONH*(YONH*(0.2*0.3Y)+SIGT+ALFA*AONH**2/(D*Y)
C                     CALCULATE P2
      IF(3*H.GT.2.0)GO TO 190
      PSI=0.3025*(1.0-0.2*AONH)*(1.0+0.5*B/Y)

      GO TO 200
  190 PSI=0.3019*(1.0+0.5*B/Y)
  200 RS=MFS*PSI*(1.0-YONH)/SIGSY
      CO=0.5*B+1.0*H
      HOLD6=MU*SIGSY*H
      IF(RS.LE.CO)GO TO 210
      IF(RS.GT.0.5*C)RS=0.5*C
      R1=HOLD6*((RS-CO)+RS*ALOG(0.5*C/RS))
      RZ=HOLD6*CO
      GO TO 220
  210 R1=HOLD6*RS*ALOG(0.5*C/CO)
      RZ=HOLD6*RS
  220 DEFL=PSI*(C-B)/2.0
      PZ=0.284*(R1+R2+RF*C*(H-Y/3.0-DEFL)/(2.0*(H-Y/3.0)))/KZ
C                     COMPARE P1 & P2
      PDHOLD=PDIFF
      PDIFF=ABS(P1-PZ)
      PER=1=100.0*PDIFF/P1
      IF(PERR1.LT.1.0)GO TO 230
      IF(MY.NE.0.AND.PDIFF.GT.PDHOLD)LFLAG=2
C                     ITERATION WITH Y/H
      IF(XFLAG.NE.1)GO TO 227
      IF(LFLAG.EQ.2)GO TO 225
      IF(MY.GT.1)GO TO 225
      YHOLD=YONH
      YONH=YONH+0.5*(1.0*PZ/P1)
      IF(MY.GT.10)YONH=(YONH+YHOLD)/2.0
      N=N+1
      MY=MY+1
      GO TO 140
C                     SEARCH FOR Y/H
```

```
      225 KFLAG=2
      226 FORM=YUNM+YDELTA
          %=%+1
          %SY=%SY+1
          GO TO 140
      227 IF(PUDIFF.LT.PUHOLD)GO TO 226
          YOMM=YOMM-YDELTA
          PDIFF=PDHOLD
          IF(KFLAG.EQ.3.OR.%SY.GT.1)GO TO 228
          KFLAG=3
          YDELTA=-YDELTA
          GO TO 226
      228 KFLAG=2
          YDELTA=0.1*YDELTA
          %SY=0
          GO TO 226
      230 PU=(PL+P2)/2000.0
          IF(A.EQ.0.0)GO TO 240
          XCHECK=BMPF/PU
C                 COMPARE X & XCHECK
          XDHOLD=XDIFF
          XDIFF=ABS(X-XCHECK)
          PERAC=ABS(100.0*XDIFF/X)
          IF(PERPR2.LT.1.0)GO TO 239
C                       ITERATION WITH X
          YDELTA=0.1
          %Y=3
          %SY=0
          KFLAG=1
          LFLAG=1
          IF(%FLAG.NE.1)GO TO 237
          IF(%.NE.0.AND.XDIFF.GT.XDHOLD)GO TO 235
          X=(X+XCHECK)/2.0
          %=%+1
          GO TO 140
C                       SEARCH FOR X
      235 %FLAG=2
      236 X=X+XDELTA
          %=%+1
          %SX=%SX+1
          GO TO 140
      237 IF(XDIFF.LT.XDHOLD)GO TO 230
          X=X-XDELTA
          XDIFF=XDHOLD
          IF(%FLAG.EQ.2.OR.%SX.GT.1)GO TO 238
          %FLAG=3
          XDELTA=-XDELTA
          GO TO 236
      238 %FLAG=2
          XDELTA=0.1*XDELTA
          %SX=3
          GO TO 236
C                 CHECK COMPRESSIVE BOUNDARY EDGE
      239 CCHECK=FP*((DP/2.0)-(DLPLP4.0))*SUCY*L.7/2.0
          CDIFF=ABS(CCM-CCHECK)
          PERAS=100.0*CDIFF/CCM
          IF(PERR3.LT.1.0)GO TO 240
          I=I+1
          GO TO 135
C                 PRINT OUTPUT
      240 IF(IFLAG.EQ.2)GO TO 242
          WRITE(6,241)
      241 FORMAT(1X,'NEGLECTING BOUNDARY REST<AINTS  (LST<AINT FACTOR
         1',F4.2,'F/IE.0)')
          WRITE(6,244)BF
      244 FORMAT(1X,'BOUNDARY FORCE  ',F12.2,' LBS/IN')
          WRITE(6,245)BM
      245 FORMAT(1H ,'BOUNDARY MOMENT ',F12.2,' IN LBS/IN')
      246 IF(Y.GT.LB.1.0)GO TO 250
          IF(TRACE)WRITE(6,3601)M,PER41
          IF(TRACE)WRITE(6,3602)M,PER42
          IF(TRACE)WRITE(6,3603)M,PER43
          IF(TRACE)WRITE(6,247)
      247 FORMAT(1H ,'Y/M CALCULATED TO BE GREATER THAN 1.0   EXIT')
          IF(IFLAG.EQ.1)GO TO 40
          GO TO 381
      250 IF(TRACE)WRITE(6,2501)YOMM
      2501 FORMAT(1H ,'Y/M = ',F5.3)
          IF(TRACE.AND.IFLAG.EQ.2)WRITE(6,2502)
      2502 FORMAT(1H ,'R = ',F6.3)
          RSOMM=RS/M
          IF(TRACE)WRITE(6,9016)SOMM
      9016 FORMAT(1H ,'RS/M = ',F5.3)
          COMM=CO/M
          IF(TRACE)WRITE(6,3303)COMM
      3303 FORMAT(1H ,'CO/M = ',F5.3)
          IF(TRACE)WRITE(6,3104)PSI
      3104 FORMAT(1X,'PSI=',F7.5,'RADIANS')
          WRITE(6,3111)DEFL
      3111 FORMAT(1X,'DEFLECTION =',F10.5,' INS')
          ALFA=ATAN(TANA)
          IF(TRACE)WRITE(6,3153)ALFA
      3153 FORMAT(1H ,'ALFA = ',F5.3,' RADIANS')
          IF(TRACE.AND.IFLAG.EQ.2)WRITE(6,317)
      317 FORMAT(1H0,26X,'(%AXIMISED)')
          IF(IFLAG.EQ.2)GO TO 394
          IF(TRACE)WRITE(6,3403)P,PER41
      3403 FORMAT(1H0,'NO. OF CYCLES  ',I2,5X,'ERROR  ',F9.3,'%')
          %=0
      325 WRITE(6,3331)PU
      3331 FORMAT(1X,'ULTIMATE PUNCHING LOAD PU = ',F10.2,' KIPS')
          PHOLD=PU
          DEP%=1.2*PU
          WRITE(6,3431)DEP%
      3431 FORMAT(1X,'ALLOWING FOR DOWEL EFFECT 1.2*PU = ',F10.2,
         1' KIPS')
          DE=0.2*PU
          IFLAG=2
          GO TO 133
      350 IF(TRACE)WRITE(6,3601)M,PER81
      360 FORMAT(1H0,'TOTAL NO. OF CYCLES',6X,'PCN Y/M  ',I3,7X,'ERROR ',F6.
         13,'%')
          IF(A.NE.0.0)GO TO 364
          WRITE(6,3621)
      362 FORMAT(1H0)
          GO TO 367
      364 IF(TRACE)WRITE(6,3651)M,PER42
      365 FORMAT(1H ,5X,'PUS X      ',I2.7X,'ERROR ',F6.3,'%')
          IF(TRACE)WRITE(6,3661)I,PER43
      366 FORMAT(1H ,5X,'PIN CUM    ',I2.7X,'ERROR ',F6.3,'%')
      367 PINC=100.0*(PU-PHOLD)/PHOLD
          WRITE(6,3703)PU,PINC
      370 FORMAT(1X,'ULTIMATE PUNCHING LOAD PU = ',T45,F6.2,
         1' KIPS   (INCREASE  ',F6.1,'%)')
          DEP=PU+DE
          PINC=100.0*(DEP-UEPN)/DEPN
          WRITE(6,3831)DEP,PINC
      3831 FORMAT(1X,'ALLOWING FOR ORIGINAL DOWEL EFFECT PU = ',T45,F6.2,
         1' KIPS   (INCREASE  ',F6.1,'%)')
      3311 IF(IFF.LT.1.0)WRITE(6,3621)
          GO TO 133
C
      399 IF(.NOT.INTAIV) GO TO 999
          WRITE(6,700)
      700 FORMAT(1X,'IS IT A NEW PROBLEM ?')
          READ(5,900)INDATA
          CALL PREAD
          IF(NANG.EQ.0) GO TO 400
          GO TO 610
C
      999 RETURN
          END
```

Example of Program — DECKBFU

Input and Echo Print of Input

```
                                                                    
                                                                    
                                                                    
```

```
  237 IF(XDIFF.LT.XDMOLD)GO TO 236
      X=X-XDELTA
      XDIFF=XDMOLD
      IF(MFLAG.EQ.3.OR.NSX.GT.1)GO TO 238
      MFLAG=3
      XDELTA=-XDELTA
      GO TO 236
  238 MFLAG=2
      XDELTA=0.1*XDELTA
      NSX=0
      GO TO 236
C                 PRINT OUTPUT
  240 IF(IFLAG.EQ.2)GO TO 242
      WRITE(6,241)
  241 FORMAT(1X,'NEGLECTING BOUNDARY RESTRAINTS'/1X,30('*'))
      IF(MU.EQ.0.0)GO TO 325
      GO TO 246
  242 WRITE(6,243)
  243 FORMAT(1X,'CONSIDERING BOUNDARY RESTRAINTS'/1X,31('*'))
      WRITE(6,244)BF
  244 FORMAT(1X,'BOUNDARY FORCE  ',F10.1,' LBS/IN')
      WRITE(6,245)BM
  245 FORMAT(1H ,'BOUNDARY MOMENT ',F10.1,' IN LBS/IN')
  246 IF(YONM.LE.1.0)GO TO 250
      IF(TRACE)WRITE(6,3601M,PERR1
      IF(TRACE)WRITE(6,3651N,PERR2
      WRITE(6,247)
  247 FORMAT(1X,' ***ERROR*** Y/H CALCULATED TO BE GREATER THAN 1.0')
      IF(IFLAG.EQ.2)GO TO 40
      IFLAG = 2
      GO TO 133
C
  250 IF(TRACE)WRITE(6,280)YONM
  280 FORMAT(1H ,'Y/H = ',F5.3)
      IF(TRACE.AND.IFLAG.EQ.2)WRITE(6,285)X
  285 FORMAT(1H ,'X = ',F6.3)
      RSONM=RS/M
      IF(TRACE)WRITE(6,290)RSONM
  290 FORMAT(1H ,'RS/M = ',F5.3)
      COONM=CO/M
      IF(TRACE)WRITE(6,300)COONM
  300 FORMAT(1H ,'CO/M = ',F5.3)
      IF(TRACE)WRITE(6,310)PSI
  310 FORMAT(1X,'PSI=',F7.5,'RADIANS')
      WRITE(6,311)DEFL
  311 FORMAT(1X,'DEFLECTION=',1PG6.5,' INS')
      ALFA=ATAN(TANA)
      IF(TRACE)WRITE(6,315)ALFA
  315 FORMAT(1H ,'ALFA = ',F5.3,' RADIANS')
      IF(TRACE.AND.JFLAG.EQ.2)WRITE(6,317)
  317 FORMAT(1H0,24X,'(MAXIMISED)')
      IF(IFLAG.EQ.2)GO TO 350
      IF(TRACE)WRITE(6,320)M,PERR1
  320 FORMAT(1H0,'NO. OF CYCLES  ',I2,5X,'ERROR  ',F9.3,'%'/)
      M=0
  325 WRITE(6,330)PU
  330 FORMAT(1X,'ULTIMATE PUNCHING LOAD PU = ',T45,F6.2,' KIPS')
      PHOLD=PU
      DEPN=1.2*PU
      WRITE(6,340)DEPN
  340 FORMAT(1X,'ALLOWING FOR DOWEL EFFECT 1.2*PU = ',T45,F6.2,
     1' KIPS'/)
      DE=0.2*PU
      IFLAG=2
      GO TO 133
  350 IF(TRACE)WRITE(6,360)M,PERR1
  360 FORMAT(1H0,'TOTAL NO. OF CYCLES'/63,'FOR Y/H  ',I3,7X,'ERROR ',F6.
     13,'%')
      IF(X.NE.0.0)GO TO 364
      GO TO 367
  364 IF(TRACE)WRITE(6,365)N,PERR2
  365 FORMAT(1H ,5X,'FOR X  ',I2,7X,'ERROR ',F6.3,'%'/)
  367 PINC=100.0*(PU-PHOLD)/PHOLD
      WRITE(6,370)PU,PINC
  370 FORMAT(1X,'ULTIMATE PUNCHING LOAD PU = ',T45,F6.2,
     1' KIPS   (INCREASE  ',F6.1,'%)')
      DE=PU+DE
      PINC=100.0*(DEP-DEPN)/DEPN
      WRITE(6,380)DEP,PINC
  380 FORMAT(1X,'ALLOWING FOR ORIGINAL DOWEL EFFECT PU =',T45,F6.2,
     1' KIPS   (INCREASE  ',F6.1,'%)')
      IF(INTRIV)GO TO 391
      GO TO 40
  390 IF(.NOT.INTRIV) GO TO 999
  391 WRITE(6,136)
      WRITE(6,700)
  700 FORMAT(1X,'IS IT A NEW PROBLEM ?')
      READ(5,900)INCATA
      CALL PREAD
      IF(NARG.EQ.0)GO TO 400
      GO TO 610
  999 RETURN
      END
```

Example of Program — DECKBFK

Input and Echo Print of Input

```
 ENTER PROGRAM NUMBER
 ?1
 DO YOU WANT PRINTOUT OF STANDARD DATA FILES?
 IF 'YES' PUSH 'RETURN' KEY - IF 'NO' ENTER 'NO'
 ?NO
 ENTER TEST NO
 ?1
 ENTER OVERALL THICKNESS OF SLAB (INS)
 ?7.5
 ENTER EFFECTIVE THICKNESS OF SLAB (INS)
 ?6
 ENTER EQUIVALENT DIAMETER OF LOADED AREA (INS)
 ?20
 ENTER EQUIVALENT DIAMETER OF SLAB (INS)
 ?96
 ENTER CUBE INNER STRENGTH OF CONCRETE (PSI)
 ?4000
 ENTER RATIO OF REINFORCEMENT
 ?.006
 ENTER YIELD POINT STRESS OF STEEL (PSI)
 ?60000
 ENTER BOUNDARY MOMENT (IN-LBS IN)
 ?2960
 ENTER BOUNDARY FORCE (LBS IN)
 ?2406
 ENTER MODULUS OF ELASTICITY OF STEEL (PSI)
 ?29000000

 * TEST DATA *
 ************
 1.OVERALL THICKNESS OF SLAB              7.500 INS
 2.EFFECTIVE THICKNESS OF SLAB            6.000 INS
 3.EQUIVALENT DIAMETER OF LOADED AREA    20.000 INS
 4.EQUIVALENT DIAMETER OF SLAB           96.000 INS
 5.CUBE INNER STRENGTH OF CONCRETE     4000.0 PSI
 6.RATIO OF REINFORCEMENT                 0.0060
 7.YIELD POINT STRESS OF STEEL        60000.0 PSI
 8.BOUNDARY MOMENT                     2960.00 IN-LBS IN
 9.BOUNDARY FORCE                      2406.00 LBS IN
 10.MODULUS OF ELASTICITY OF STEEL  29000000.0 PSI

 IS THE INPUT CORRECT ?
```

Typical Output

```
 * NEGLECTING BOUNDARY RESTRAINTS *
   ********************************
 DEFLECTION=                                  .361 INS
 ULTIMATE PUNCHING LOAD PU =                 129.12 KIPS
 ALLOWING FOR DOWEL EFFECT 1.2*PU =          154.95 KIPS

 * CONSIDERING BOUNDARY RESTRAINTS *
   *********************************
 BOUNDARY FORCE      2406.0 LBS IN
 BOUNDARY MOMENT     2960.0 IN LBS IN
 DEFLECTION=                                  .362 INS
 ULTIMATE PUNCHING LOAD PU =                 129.12 KIPS   INCREASE   281.3%
 ALLOWING FOR ORIGINAL DOWEL EFFECT PU =     154.94 KIPS   INCREASE   234.4%

                    ********************

 * IS A NEW PROBLEM ?
 NO
```

Require to Make a Change in the Ratio of Reinforcement Parameter

```
 ENTER EST NO.AND CORRECTED VALUE- IF NO MORE CHANGES   PUSH RETURN
 ?0,0
 ENTER EST NO.AND CORRECTED VALUE- IF NO MORE CHANGES   PUSH RETURN
```

Echo Print of Input

```
 * TEST DATA *
 ************
 1.OVERALL THICKNESS OF SLAB              7.500 INS
 2.EFFECTIVE THICKNESS OF SLAB            6.000 INS
 3.EQUIVALENT DIAMETER OF LOADED AREA    20.000 INS
 4.EQUIVALENT DIAMETER OF SLAB           96.000 INS
 5.CUBE INNER STRENGTH OF CONCRETE     4000.0 PSI
 6.RATIO OF REINFORCEMENT                 0.0060
 7.YIELD POINT STRESS OF STEEL        60000.0 PSI
 8.BOUNDARY MOMENT                     2960.00 IN-LBS IN
 9.BOUNDARY FORCE                      2406.00 LBS IN
 10.MODULUS OF ELASTICITY OF STEEL  29000000.0 PSI

 IS THE INPUT CORRECT ?
```

Output

```
'FLECTING BOUNDARY RESTRAINTS
*****************************
'FLECTION=                          0.617 INS
ULTIMATE PUNCHING LOAD PU =          92.67 KIPS
WAITING FOR DOWEL EFFECT 1.2*PU =   111.20 KIPS

CONSIDERING BOUNDARY RESTRAINTS
*******************************
BOUNDARY FORCE     3686.0 LBS/IN
BOUNDARY MOMENT    2960.0 IN LBS/IN
'FLECTION=                          0.336 INS
ULTIMATE PUNCHING LOAD PU =         206.33 KIPS   (INCREASE   122.7%)
WAITING FOR ORIGINAL DOWEL EFFECT PU =  224.87 KIPS   (INCREASE   102.2%)
                          **********
```